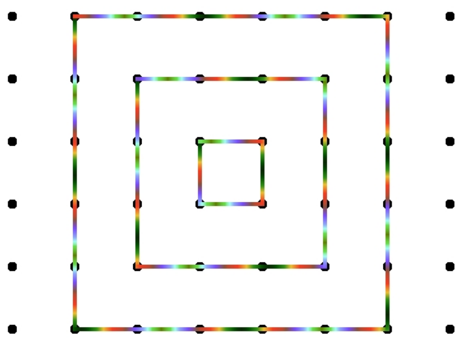
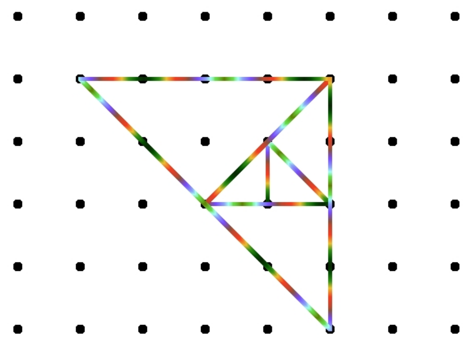
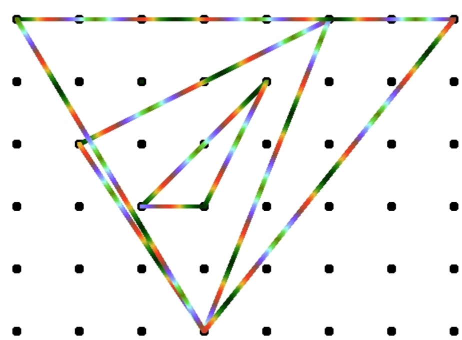
WEEKLY PROBLEM NOVEMBER 29 TO DECEMBER 5 2009

PICK'S THEOREM

This problem is meant to discover the statement of Pick's Theorem. First compute the area of each square in the first figure below. Note that the area is in terms of the units used in the grid. For example, the smallest square has area 1. Next count the number of interior lattice points and the number of boundary lattice points. For example, the smallest square has no interior lattice points, and has 4 boundary lattice points. Do the same for all the figures below remembering that the area of a triangle is $1/2 \cdot$ base $\cdot$ height .







Can you deduce a formula in terms of the number of interior lattice points and the number of boundary lattice points that holds for all the examples you worked out? Now write down a few polygons of your own and check the results with your formula. If your formula does not hold for *any* example, then it cannot be correct. If it does hold for all the examples you try, can you think of any way to *prove* it?